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The past and future of knowledge-based growth

Holger Strulik · Klaus Prettner · Alexia Prskawetz

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Abstract This paper consolidates two previously disconnected literatures. It integrates R&D-based innovations into a unified growth framework with micro-founded fertility and schooling behavior. The theory suggests a refined view on the human factor in productivity growth. It helps to explain the historical emergence of R&D-based growth and the subsequent emergence of mass education and the demographic transition. The model predicts that the erstwhile positive correlation between population growth and innovative activity turns negative during economic development. This "population-productivity reversal" explains why innovative modern economies are usually characterized by low or negative population growth. Because innovations in modern economies are based on the education of the workforce, the medium-run prospects for future economic growth—when fertility is going to be below replacement level in virtually all developed countries—are better than suggested by conventional R&D-based growth theories.

Keywords R&D · Productivity · Fertility · Human capital · Demographic transition

Some ideas of the present paper were first available in Strulik et al. (2010) but in this forerunner version of the present paper the theory was not yet embedded into a unified growth context.

H. Strulik (✉) · K. Prettner
Department of Economics, University of Goettingen, Platz der Goettinger Sieben 3, 37073 Goettingen, Germany
e-mail: holger.strulik@wiwi.uni-goettingen.de

K. Prettner
e-mail: klaus.prettner@wiwi.uni-goettingen.de

A. Prskawetz
Institute of Mathematical Methods in Economics, Vienna University of Technology, Vienna, Austria

A. Prskawetz
Vienna Institute of Demography, Austrian Academy of Sciences, Vienna, Austria

A. Prskawetz
Wittgenstein Centre for Demography and Global Human Capital, Vienna, Austria
e-mail: afp@econ.tuwien.ac.at

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1 Introduction

A characteristic feature of economic theories designed to explain the performance of human societies over the very long run is that they are emphasizing the interaction between economic and demographic variables as crucial for our understanding of economic development. Unified Growth Theory suggests that the transition from stagnation to growth has been an inevitable by-product of the process of development (Galor 2005, 2011). The demo-economic history of countries or regions is conceptualized as subdivided into a Malthusian era and a modern era, and a transition between these regimes. During the Malthusian era population size interacts positively with the rate of technological progress, the population expands gradually and income is almost constant at a low level. Ultimately, increasing productivity makes mass education worthwhile such that income constrained parents prefer to have less and better educated children and the fertility transition sets in. Accelerating human capital formation contributes further to technological progress such that economic growth increases gradually while population growth declines as the economy converges towards steady growth in the modern era.

So far the literature has not focussed on the late phase of the fertility transition and on the phenomenon that fertility does not stop declining when it reaches replacement level. Actually, however, the total fertility rate (TFR) fell below replacement level in the 1970s in Europe and Japan, in the 1980s in North America and Australia, and in the 1990s in the Asian Tiger countries (Bongaarts 2001; Strulik and Vollmer 2013). It is now below replacement level in all 50 European countries but Turkey (where it is at 2.15) and in more than 80 countries in the world (UN 2011). Table 1, compiled from UN (2011), shows the most recently observed TFR for the G-8 countries, i.e. those countries that we usually associate with production at the "frontier of technological knowledge" (Aghion and Howitt 2009). In every country that contributes substantially to innovation-based, R&D-driven growth the TFR is below replacement level (i.e. TFR below 2.1).

Among the developed countries the U.S. is unique in displaying a TFR close to replacement level. Table 2, compiled from U.S. National Center for Health Statistics (2010), shows that this achievement originates solely from the high TFR of the Hispanic part of the population. The TFR of non-Hispanic whites (1.83), for example, is close to that of their European forefathers. Assuming that fertility behavior of immigrants is at least partly rooted in the fertility norms of

Table 1 TFR for the G-8: 2005–2010

USA	2.07	France	1.97
U.K.	1.83	Canada	1.65
Italy	1.38	Germany	1.36
Russia	1.44	Japan	1.32

Table 2 TFR USA 2008

Non-Hispanic white	1.83
Asian-American	2.05
Black	2.11
Hispanic	2.90

the country of origin we expect fertility of the Hispanic population in the U.S. to fall below replacement level with ongoing fertility transition in the countries of origin. Some Latin American countries (e.g. Chile, Brazil, Cuba) already display fertility below replacement and for other countries this seems to be likely in the future. In 2008 the United Nations predicted according to their medium-variant projection that every country converges towards a TFR of 1.85 in the medium run, i.e. a fertility pronouncedly below replacement level (UN 2008). Inspired by some recent mild recoveries of fertility the latest UN projection assumes again convergence towards replacement level, albeit with heavy undershooting; for Europe, Asia, and Latin America the TFR is predicted to remain below replacement level over the whole twenty-first century (UN 2011).

According to conventional theories of R&D-based growth, the fact that the population is declining entails a grim economic outlook for modern economies. Models of the first generation (Romer 1990; Aghion and Howitt 1992) provide the result that growth of aggregate productivity (TFP) is linearly related to population size. Thus, a declining population implies vanishing growth of productivity and income per capita. According to models of the second generation (Jones 1995; Kortum 1997; Segerstrom 1998), TFP growth at the steady state is linearly related to population growth, implying the prediction that the long-run growth rate of income per capita is zero when the population is stationary.[1]

R&D Models of the third generation (Peretto 1998; Young 1998; Howitt 1999) eliminate the strong tie between population and productivity by considering that growth in product variety reduces the effectiveness of product quality R&D. These models predict that, at the steady state, growth of vertical innovations is independent from population growth while growth of horizontal innovations is still determined by population growth. The number of available products grows at the same rate as the population. Depending on the specific relationship between product variety and aggregate productivity, growth of productivity is still positively associated with population growth (as in Howitt 1999) or, at the steady state, independent from population growth (as in Peretto 1998). In the latter case a positive association between the two variables is retained as a phenomenon of transitional dynamics. During the demographic transition declining population growth is associated with declining productivity growth.[2]

One could argue that R&D-based growth theory does not apply to single countries but to larger markets because ideas can cross borders. It is thus important to notice that fertility below replacement level is not an abnormal feature of some strange country but globally observed in almost every country that could be associated with an advancement of world technology through R&D-based growth.

Against this background it is reassuring to know that for contemporaneous economies there is little empirical support for a positive association between population growth and productivity growth. In fact, in Sect. 2 we argue that the data supports the opposite. Productivity growth is negatively associated with population growth during the twentieth century, across countries as well as over time. Motivated by the fact that the available R&D-based growth theory cannot explain this phenomenon we propose a novel theory, which provides a refined view on the human factor in productivity growth.

The new theory integrates R&D-based growth into a unified growth framework and consists of three main elements. Firstly we argue that it is not the sheer number of workers (L)

[1] A quantitative exercise employing the conventional R&D-based growth model (Jones 2002) expects R&D-based growth to converge to 0.06 assuming—in contrast to the UN population predictions—that the labor force in the G-5 countries continues to grow at 1.2 annually.

[2] Ha and Howitt (2007) as well as several papers by Jakob Madsen (e.g., 2008) investigate empirically whether TFP growth is better explained by R&D expenditure per capita or by aggregate growth of R&D inputs and establish this way supportive evidence for R&D models of the third generation.

that propels the creation of ideas and the advancement of productivity but the total amount of knowledge embodied in these workers, i.e. aggregate human capital (H). The most intuitive aggregation is probably that total human capital is given by human capital per worker h times the number of workers ($H = h \cdot L$). Utilizing this notion of human capital and endogenizing the incentive to acquire it through costly schooling, a couple of papers have already demonstrated that human capital growth can take over the role of population growth in R&D-based growth models by predicting that productivity growth can be sustained with constant or declining population as long as human capital is accumulated rapidly enough; see, among others, Funke and Strulik (2000), Dalgaard and Kreiner (2001), Strulik (2005), Grossmann (2010). Yet the available literature leaves the "population-productivity reversal" unexplained. Notwithstanding the consideration of human capital it still predicts that population growth is positively associated with innovative activity.

In order to account for the reversal in the association between population growth and innovative activity we, secondly, introduce an interaction of quantity and quality of the workforce into R&D-based growth theory. The interaction originates from a micro-founded trade-off that allows parents to substitute child quality for child quantity such that h rises and L falls. The child quantity quality trade-off is also at the center of unified growth theory. Our paper adds to the established literature an explicit mechanism through which population scale and quality have a differential effect on technological change at different stages of development. Also, R&D-driven productivity growth has not yet been investigated in a unified growth framework.[3]

The third element is to discover the mechanism according to which the quantity-quality trade-off leads to more aggregate human capital, i.e. to explain why h rises more strongly than L falls. To see how the mechanism works, consider a unit increase of education expenditure in company with a unit reduction of fertility such that total child expenditure remains constant. Such a one-to-one quantity-quality substitution is not neutral. It sets free parental time because less time is needed for child rearing. The time gained can be used to earn additional income, which in turn is spent on consumption and on children's education. This means the original one-to-one substitution leads actually to a situation where children's education increases by more than fertility falls. As a consequence, aggregate human capital increases. Since this mechanism is based on interaction in the budget constraint (and not on specifications of the utility function) we are confident that our results hold with some generality.

The model explains the following evolution of knowledge-based growth. For most of human history, productivity growth originated from learning-by-doing and was thus positively associated with population growth. Eventually, through growing L, market R&D became worthwhile. After this first industrial revolution the economy expanded through R&D-based growth and high population growth (from today's perspective) while productivity and population were still positively associated. Eventually, however, human capital became precious enough for mass-education to become worthwhile. A demographic transition was initiated during which subsequent generations of parents decided to have fewer children and to educate them better. During the fertility transition a quantity-quality mechanism operated as explained above: the increase of individual human capital endowments

[3] For unified growth theory see, among others, Galor and Weil (2000), Galor and Moav (2002), Galor and Moav (2004), Galor et al. (2009), De la Croix and Doepke (2003), De la Croix and Doepke (2004), Moav (2005), Galor (2005), and Galor (2011). A few articles have integrated endogenous fertility into an R&D-based growth framework, notably Jones (2001), Connolly and Peretto (2003) and Growiec 2006), but the interaction with education and the quantity-quality trade-off has not been investigated. Since we embarked on this project Chu et al. (2013) also investigated human capital and fertility in an R&D-based growth framework albeit with a completely different focus, namely the impact of patent protection on human capital formation and fertility.

overcompensated the associated decline of raw labor such that aggregate human capital and income per capita started to grow at unprecedented rates while population growth declined.

Altogether this means that our theory explains a productivity reversal. Before the onset of the fertility transition, increasing fertility and population growth contributed positively to productivity growth. The "old" R&D-based growth theory is thus not completely abandoned. It is still present as a temporary, intermediate phase during which R&D is worthwhile but the fertility transition has not yet been initiated. When the child quantity-quality trade-off becomes operative, productivity growth becomes negatively associated with population growth, as observed during and after the twentieth century. This negative association is bi-causal and holds irrespective of family size, that is, in particular, also for fertility below replacement level. The refined model thus provides a much brighter outlook for the future of R&D-based growth in contemporary societies. It predicts high growth of income and productivity when fertility is close to or below replacement level.

The paper is organized as follows. The next section explores the empirical association between population growth and productivity growth. Section 3 sets up the R&D-based growth model. Section 4 analyzes the balanced growth path and provides the results from comparative static analysis. Section 6 calibrates the model to capture the long-run development of an aggregate G-7 economy. It demonstrates that the theory can correctly predict the productivity reversal and matches other demographic and macroeconomic aggregates (fertility, education, R&D labor input and GDP per capita) reasonably well. Furthermore, the model offers novel insights about the onset of innovation based growth (the first Industrial Revolution) and mass education driven growth (the second Industrial Revolution). We then use the model to predict economic growth for the twenty-first and twenty-second century given the high-, medium-, and low-fertility projections of the UN. We conclude with a speculative outlook for future economic development.

2 The population-productivity reversal

For most of human history the growth of productivity and population were presumably positively associated. Productivity growth allowed the subsistence of larger families (e.g. Ashraf and Galor 2011) and a larger population meant a larger number of tinkerers producing more ideas (Kremer 1993). We are not challenging this role of population growth for productivity growth in *pre-modern* human history. But we argue that in *modern* times, specifically after the onset of the fertility transition and of R&D as a market activity, the erstwhile positive association between population and productivity growth turned negative. Across countries, R&D-driven productivity growth is high in countries with low or negative population growth. Over time, within the group of countries at the knowledge frontier, productivity growth is increasing when fertility is decreasing. These are the observations which are hard to square with conventional R&D-based growth theory.

For the twentieth century a negative association between population growth and income or income growth has already been documented by several studies (e.g. Brander and Dowrik 1994; Kelley and Schmidt 1995; Ahituv 2001; Li and Zhang 2007; Herzer et al. 2012). The association between average annual growth of population and TFP growth across countries for the period 1950–2000 is shown in Fig. 1 (calculated from the data in Baier et al. 2006). Across all countries for which data is available (identified in the Figure by blue crosses) the simple correlation is clearly negative (see Bernanke and Guerkaynak, 2001, for a similar finding).

As already mentioned in the Introduction, R&D-based growth is probably not well measured at the country-level because the locally created knowledge diffuses internationally.

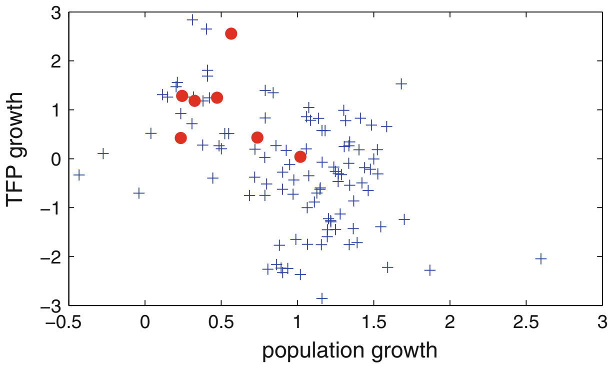

Fig. 1 Population growth versus TFP growth 1950–2000. Growth rates are average annual growth rates 1950–2000 calculated from Baier et al. (2006). *Blue crosses*: all available countries, *red dots*: G-7 countries (Color figure online)

But knowledge spillovers decline with distance and are smaller across countries than within countries (Jaffe et al. 1993; Keller 2002; Bottazzi and Peri 2003). If conventional R&D-based growth theory is right we would expect that at least some of the high TFP growth generated in countries where population growth was high to be visible in the data. But Fig. 1 seems to suggest the opposite.

Acknowledging that developing countries—i.e. the places where usually population growth is higher—do not much advance TFP growth by market R&D activities we may focus just on the G-7 countries. For these rich and fully developed countries, which accounted for 84 percent of worldwide R&D spending in 1995 (Keller 2009), and which have presumably pushed the world technology frontier, a positive association between population growth and TFP growth remains invisible (red dots in Fig. 1).

In order to explore the association between TFP and population growth a bit further, we constructed TFP growth rates, growth rates of the labor force and education indicators in ten year steps between 1950 and 2000 for a sample of 142 countries using data from Baier et al. (2006) and the World Bank (2012). This allowed us to control for country- and time-specific fixed effects. The estimated model reads

$$g_{i,t} = \beta_0 + \beta_1 x^c_{i,t-1} + \beta_2 x_{i,t} + \epsilon_i + \kappa_t + u_{i,t}, \quad (1)$$

in which $g_{i,t}$ represents average TFP growth in country i between time t and $t-1$. The matrix $x^c_{i,t-1}$ contains as control variables the degree of openness and the investment share, each lagged by one time period, while the matrix $x_{i,t}$ contains different measures of work force growth between time t and $t-1$ or different measures for tertiary education in county i at time $t-1$ and the respective growth rate of tertiary education. The variable ϵ_i denotes country specific fixed effects, κ_t are time specific fixed effects, $u_{i,t}$ is the error term and β_0, β_1 and β_2 are the coefficients to be estimated. The results are reported in Table 3 for the total sample referred to as "WORLD" and for the G-7 countries. Since population growth affects the growth of the work force with a delay of one generation, we use lag two (corresponding to a shift of 20 years) when approximating work force growth by population growth. In case of educational measures, we use one time lag to reduce the problem of reverse causality.

As Table 3 documents, the central qualitative result of a negative correlation between TFP growth and population growth is robust to model specifications with respect to country- and time-specific fixed effects as well as with respect to controlling for the openness of a country

Table 3 TFP growth, population growth and education (1950–2000)

	WORLD	G-7	WORLD	G-7	WORLD	G-7	WORLD	G-7
Openness (L1)	0.008	−0.039	0.029	0.076	0.006	0.053	0.008	0.028
	(0.006)	(0.026)	(0.009)***	(0.017)***	(0.010)	(0.028)	(0.009)	(0.033)
Investment share (L1)	−0.005	−0.064	−0.018	0.057	−0.017	0.045	−0.019	0.035
	(0.006)	(0.029)*	(0.010)*	(0.026)*	(0.009)*	(0.025)	(0.008)**	(0.022)
Labor force growth	−1.410	−1.557						
	(0.292)***	(0.642)*						
Population growth (L2)			−0.488	−0.394				
			(0.386)	(0.300)				
Growth of tertiary education (L1)					0.004	0.010		
					(0.004)	(0.002)***		
Tertiary education (L1)					0.003	0.000		
					(0.001)**	(0.001)		
Growth years of education (L1)							0.004	0.010
							(0.004)	(0.003)**
Years of education (L1)							0.052	0.014
							(0.022)**	(0.018)
R^2	0.19	0.64	0.11	0.51	0.10	0.70	0.11	0.68
OBS	455	26	289	20	303	20	311	20
Country fe	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fe	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Dependent variable: TFP growth. Standard errors are reported in parentheses. * indicates significance at the 10 % level, ** indicate significance at the 5 % level and *** indicate significance at the 1 % level. Tertiary education is the fraction of the population above the age of 25 with tertiary education and years of education are the average years of tertiary education of the population above the age of 25. L1 and L2 refer to the first and second lag and its investment share of GDP. The correlation between TFP growth and measures for workforce growth tends to be negative for the whole sample and for the G-7. Consequently, we do not find supporting evidence for a positive correlation between TFP growth and population growth. Given the possibility of reverse causality, these exercises are, of course, not sufficient to reject a potential causal positive impact of population growth on TFP growth.[4] But it is hard to come up with a mechanism that would be so strong that it overturns the allegedly positive impact such that it becomes invisible in the data.

The second part of Table 3 shows that TFP growth is positively associated with tertiary education, measured either by the fraction of the population with tertiary education and its growth rate or by average years of tertiary education and its growth rate. The result confirms the view that productivity growth is advanced by well educated scientists and engineers. In conjunction with the results with respect to population growth it points towards a quality-quantity trade-off. Higher productivity is achieved by having a smaller but better educated workforce. The quality-quantity trade-off establishes a causal link between population growth (fertility) and education, which prevents to have both variables simultaneously in the regression. We discuss further implications of this feature later on with the help of the theoretical model.[5]

We next expand the time window to cover also a period before the onset of the fertility transition in today's fully developed countries. Related exercises of quantitative economic history have singled out the G-5 countries as the most important drivers of the world technology frontier (Jones 2002; Ha and Howitt 2007). Since the data is available we extend the set of countries a bit towards the G-7 in favor of more general results (i.e. US, UK, Germany, France, Japan, Canada, and Italy). Anyway, the pictures that we obtain for the G-7 and G-5 look very similar. From the individual countries' time series we compute population-weighted average time series for population growth and TFP growth during the 1860–2000 period. We smooth the obtained series of annual average growth rates using the Hodrick–Prescott (1997) filter and a smoothing parameter of 100 in order to uncover the long-run trend.

The panel on the left hand side of Fig. 2 shows the average trend growth rates for the G-7 countries. The blue (solid) line shows TFP growth and is scaled along the axis on the left hand side. The green (dashed) line shows population growth and is scaled along the axis on the right hand side. At the beginning, before the onset of the fertility transition, population growth was relatively high (from today's perspective) and TFP growth was very low. At the end, population growth is small but still positive due to falling old age mortality, and TFP growth is relatively large (from nineteenth century perspective). If we imagine today's LDCs are where the G-7 were a century ago (Dalgaard and Strulik 2013), it is easy to rationalize the negative correlation between population growth and TFP growth observed across countries in the larger sample.

Until the year 1920, which roughly coincides with the onset of the fertility transition in the G-7 countries, population growth and TFP growth are jointly rising, as observed by

[4] As a robustness check we also estimated a dynamic panel regression using different GMM estimators. We found again a significantly negative association between productivity growth and population growth and a significantly positive association between productivity growth and our measures for education. However, since the lagged dependent variable was never significantly different from zero, our preferred estimates are those in Table 3.

[5] The model predicts that along the steady state productivity growth is positively associated with human capital growth. Off the steady state, the level of human capital may play a role as well. Accordingly we find a significantly positive level effect for the whole sample, containing countries that are presumably far off their steady state. For the G-7, a group of countries which are presumably closer to the steady state, we a find significantly positive correlation between growth rates and the level of human capital turns insignificant.

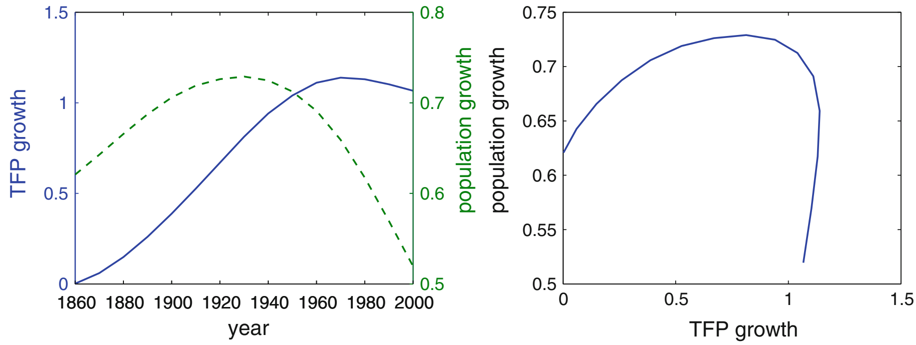

Fig. 2 Population growth and TFP growth: G-7 countries, 1860–2000. *Data sources*: see Appendix. Time series smoothed with Hodrick Prescott filter (factor 100). *Left*: *solid (blue) line*: TFP growth; *dashed (green)* line population growth. *Right*: The implied trajectory for population growth–TFP growth (Color figure online)

Kremer (1993).[6] Afterwards, fertility declines and TFP continues to rise further, reaching an unprecedented high in the 1970s, roughly at the time when fertility falls below replacement level in most of the G-7 countries. Afterwards, the Hodrick–Prescott filter identifies mildly decreasing TFP growth while population growth continues to decline. The panel on the right hand side of Fig. 2 shows the semicircle described by the joint evolution of population growth and TFP growth with a maximum reached in 1920, around the time of the onset of the fertility transition.

During the period of investigation average years of schooling in the G-7 increased continuously from 0.57 years in 1860 to 11.9 years in 2000 (see Appendix for computational details). In 1920, at the time of the fertility reversal, G-7 children had on average 4.0 years of eduction. One attempt to square conventional R&D-based growth theory with these facts could be to argue for long gestation lags of human capital. In that case the high TFP growth at the end of the twentieth century could have been accomplished by relatively old scientists and engineers of the relatively large but less well educated cohorts born 1920-1950. The alternative—that we propose below—is to acknowledge a child quantity-quality trade-off. In that case it is possible that the high TFP growth rates at the end of the twentieth century have been accomplished by relatively small cohorts of young and well educated scientists and engineers born after the 1950s.

3 The model

3.1 Households

Consider an economy populated by three overlapping generations, children, young adults, and old adults. Children consume the provisions received by their parents and old adults consume their savings plus interest. Young adults supply one unit of labor and decide how to split their income between current consumption and future consumption, how many children they want to have, and how much they want to spend on their children's education.

In order to derive the main results conveniently and to get explicit solutions, we make a number of simplifying assumptions. Each unisex household consists of one parent (which

[6] Reher (2004) identifies the onset of the fertility decline for the G-7 as 1900 for Germany and France, 1910 for England, 1915 for Canada, 1925 for the US and Italy, and 1950 for Japan.

avoids to tackle matching problems), there is no explicit consideration of mortality (which avoids problems of uncertain survival), children are a continuous number (which avoids problems of indivisibility), and the motive for child expenditure is non-operational (which avoids problems of maximizing dynastic value functions). This means that parents' motivation to spend on children's education is not driven by the anticipation of the increase of children's utility caused by this expenditure but by a "warm glow" of giving (Andreoni 1989) or the desire for having "higher quality" children (Becker 1960).

To be specific let c_t^1 and c_t^2 denote consumption of the young and old in period t. The currently young face a gross interest rate net of depreciation, denoted by R_{t+1} and make a savings decision s_t, which provides future consumption $c_{t+1}^2 = R_{t+1}s_t$. A young adult's human capital is denoted by h_t and the wage per unit of human capital is denoted by w_t. Let n_t denote the number of children and τ the time cost of raising a child.[7] Children acquire a minimum informal education $\bar{e}$ by observing and imitating parents and peers at work. This knowledge (of farming or a particular trade, for example) is useful, i.e. it creates human capital, but it comes basically for free, at no educational cost. For lack of a better word we call it informal knowledge in contrast to costly formal education.[8]

To increase education beyond the informal level parents may spend e_t per child, conceptualized in the Beckerian sense as child quality expenditure. Education of the current period's children determines human capital endowment of next period's adult generation (h_{t+1}). Since the production function for human capital is given to the single adult, having expenditure on education or next period's human capital in the utility function leads to similar results. Summarizing, young adults solve the problem

$$\max_{c_t, s_t, e_t, n_t} u_t = \log c_t^1 + \beta \log(R_{t+1}s_t) + \gamma \log(\bar{e} + e_t) + \eta \log n_t$$

subject to the budget constraint $w_t h_t(1 - \tau n_t) = c_t^1 + s_t + n_t e_t$. All variables have to be non-negative. The positive parameters β, γ, and η denote the weights of future consumption, child expenditure, and family size for utility, capturing the importance of these elements relative to current consumption. In order to get a meaningful problem in which a population of positive size exists, we assume $\eta > \gamma$, which ensures that $n_t > 0$. With respect to education, however, no such logical argument can be made, implying that e_t could be positive or zero depending on whether the non-negativity constraint $e_t \geq 0$ is binding or not.

From the first order conditions we obtain the solution (2) for consumption and savings regardless of whether education is interior or at the corner.

$$c_t = \frac{1}{1+\beta+\eta} \cdot w_t h_t, \qquad s_t = \frac{\beta}{1+\beta+\eta} \cdot w_t h_t. \tag{2}$$

For child quantity and quality there exists a threshold at $z \equiv \eta\bar{e}/(\gamma\tau)$. If income falls below the threshold parents do not invest in education and focus on maximizing child quantity. In particular we obtain from the first order conditions

$$e_t = \begin{cases} 0 & \text{for } w_t h_t < z \\ \dfrac{\gamma\tau w_t h_t - \eta\bar{e}}{\eta - \gamma} & \text{otherwise} \end{cases} \tag{3}$$

[7] Following Galor (2005) n_t could be interpreted as the number of children surviving up to adulthood, implicitly assuming that child costs are only associated with surviving children.

[8] Without loss of generality (but at the expense of simplicity) informal education could itself be time varying and a function of complexity in production, which in turn is approximated by the wage rate. The proof of this claim is available on request.

$$n_t = \begin{cases} \dfrac{\eta}{(1+\beta+\eta)\tau} & \text{for } w_t h_t < z \\ \dfrac{(\eta-\gamma)w_t h_t}{(1+\beta+\eta)(\tau w_t h_t - \bar{e})} & \text{otherwise .} \end{cases} \quad (4)$$

Once income surpasses the threshold z a fertility transition is initiated: further rising income leads to declining fertility and increasing expenditure for education. While education expenditure is not bounded, fertility arrives at a lower bound as income approaches infinity.

$$\lim_{w_t h_t \to \infty} n_t = \underline{n} \equiv \frac{\eta - \gamma}{(1+\beta+\eta)\tau}. \quad (5)$$

Notice that $\underline{n}$ may fall below the replacement rate of unity. In particular, $\underline{n} < 1$ for $\tau > (\eta-\gamma)/(1+\beta+\eta)$.[9]

3.2 Education

Education e_t is transformed into human capital of the next generation of young adults via a schooling technology. A reasonable technology does not just translate expenditure into human capital but controls also for the costs of schooling. These costs can be conveniently approximated by the wage w_t, i.e. the cost of a unit of human capital of the current adult (teacher-) generation. We assume a linear schooling technology $h_{t+1} = A_E(e_t/w_t) + \bar{e}$ in which A_E signifies general productivity of schooling. Without education expenditure, human capital of the next generation consists of basic skills from informal education.[10] Inserting (3) into the schooling technology provides a simple equation of motion for human capital:

$$h_{t+1} = \frac{A_E}{\eta - \gamma}\left(\gamma \tau h_t - \frac{\eta \bar{e}}{w_t}\right) + \bar{e}. \quad (6)$$

With growing income the growth rate of human capital becomes a constant.

$$\lim_{w_t h_t \to \infty} \frac{h_{t+1}}{h_t} = \Delta_h \equiv A_E \frac{\gamma\tau}{\eta-\gamma}. \quad (7)$$

The feature that in the limit the intergenerational transmission of human capital is linear generates long-run growth, given that productivity in education A_E is sufficiently large such that $\Delta_h > 1$. It means, since human capital is measured in valuable knowledge per person, that the value of knowledge increases linearly from one generation to the next.

The most important observation of family behavior at the interior solution is the child quantity-quality trade-off. It comes in two variants: it is elicited by a change of parental preferences and it is endogenously generated by rising family income, i.e. by rising wages or rising human capital endowment.

[9] The replacement level is at unity because in theory all children become parents as adults. The empirical replacement level (Sect. 2) takes into account that not all children become parents and is thus slightly above unity.

[10] Scaling educational output by wages captures the idea that a unit of education expenditure creates less additional human capital when the costs of providing education are high. Controlling for wages is essential for stability. Otherwise human capital would grow hyper-exponentially, driven by increasing h_t *and* rising w_t. A similar control for the current state of product quality is known to be essential for stability in R&D-driven quality improvements of products, see e.g. Li (2000). Our assumption that parents spend a share of GDP on education means that the production of education uses not only human capital but also intermediate products as inputs. Without loss of generality the theory could be reduced to one in which teachers are the only input in education.

Lemma 1 (Quantity-Quality Trade-off I) *A rising desire for a large family, captured by increasing η, increases fertility and reduces next generation's human capital. A rising desire for education, captured by increasing γ increases human capital of the next generation and reduces fertility.*

Lemma 2 (Quantity-Quality Trade-off II) *Rising income, i.e. rising wages w_t or rising human capital endowment h_t reduces fertility and increases human capital of the next generation.*

The proof notes that next generation's human capital is an increasing function of education. The proof of Lemma 1 inspects the first derivatives of the interior solution of n_t and e_t with respect to γ and η in (4) and (5), and finds $\partial n_t/\partial\gamma < 0$, $\partial e_t/\partial\gamma > 0$, $\partial n_t/\partial\eta > 0$, $\partial e_t/\partial\eta < 0$. The proof of Lemma 2 observes $\partial n_t/\partial(w_t h_t) < 0$ and $\partial e_t/\partial(w_t h_t) > 0$ from the interior solution in (4) and (5).

3.3 Firms: overview

The setup of firms and markets follows closely Romer (1990) and Jones (1995). The economy consists of three sectors: The R&D-sector is perfectly competitive and employs scientists to create new ideas in the form of blueprints, manifested in patents. In the intermediate goods sector, a patent is needed as fixed input to produce a specialized capital good. Purchase of a patent allows a capital goods producer to transform one unit of raw capital, i.e. one unit of individual's savings, into one blueprint-specific machine. A perfectly competitive final goods sector uses these machines and workers to assemble a consumption aggregate.

Since the firms' side of the model—aside from the special role of human capital and the possibility of a corner solution—coincides with the Romer-Jones setup, description can be brief.

3.4 Final goods sector

The final goods sector operates the following piecewise defined Cobb-Douglas production technology

$$Y_t = B_t \left(H_t^Y\right)^{1-\alpha} \int_0^{A_t} x_t(i)^\alpha \mathrm{d}i \tag{8}$$

when there is R&D and $Y_t = A_0 B_t (H_t^Y)^{1-\alpha}$ otherwise. Here, Y_t is output, H_t^Y is employment of human capital in the final goods sector, B_t is the technological level achieved through learning-by-doing, and A_0 is a normalizing constant for output before the onset of market R&D activities. The parameter α is the capital share in final goods production, $x_t(i)$ is the amount of a certain machine i used in final goods production and A_t is the number of available differentiated inputs. Given a wage w_t per unit of human capital, and a rental price $p_t(i)$ for capital input i demand for labor and differentiated inputs fulfils

$$w_t = (1-\alpha)Y_t/H_t^Y \tag{9}$$

$$p_{i,t} = \alpha B_t \left(H_t^Y\right)^{1-\alpha} x_t(i)^{\alpha-1}. \tag{10}$$

3.5 Capital goods production

Producers of specialized inputs transform one unit of raw capital into one unit of specialized capital such that $k_t = x_t$. Operating profits of an intermediate goods producer $\pi_t(i)$ are thus given by $\pi_t(i) = p_t(i)x_t(i)k_t(i) - r_t k_t(i) = \alpha B_t (H_t^Y)^{1-\alpha} k_t(i)^\alpha - r_t k_t(i)$ in which r_t denotes the interest rate that has to be paid for individual's savings. Solving the associated problem of profit maximization given demand (10) leads to the price of $p_t(i) = p_t = r_t/\alpha$ for all types of machines so that the machine-specific index can be dropped.

Free entry into capital goods production implies that in equilibrium operating profits are covering the fixed costs of production originating from purchasing a patent. In slight deviation from the original setup and inspired by Aghion and Howitt (2009, Chapter 4) we assume that a patent holds for one period (i.e. one generation) and that afterwards, in any future period $t + 1$ the monopoly right is sold at price π_{t+1} to someone chosen at random from the currently active generation. The revenue is spent unproductively on public consumption.[11] This simplification helps to avoid intertemporal (dynastic) problems of patent holding and patent pricing while keeping the basic incentive to create new knowledge intact. Summarizing, free entry implies $\pi_t = p_t^A$ where p_t^A is the price of a blueprint.

Because capital goods are sold at the same price and demanded at equal quantities, aggregate capital is given by $K_t = A_t x_t$. Inserting this information into the production of final goods, Eq. (8) simplifies to

$$Y_t = B_t A_t^{1-\alpha} \left(H_t^Y\right)^{1-\alpha} K_t^\alpha. \tag{11}$$

3.6 Knowledge production

We consider two forces driving the evolution of knowledge and aggregate productivity. First, A_t is driven by the development of new products through market R&D and, second, B_t rises through learning-by-doing. Standard R&D-based growth theory usually neglects learning-by-doing because it focusses solely on modern economies. Here, within a unified growth setting, which encompasses centuries or millennia of economic development, we need a force to drive productivity growth before the onset of market R&D. In line with Kremer (1993), Galor and Weil (2000), and Galor (2005) we assume that these learning-by-doing activities depend positively on the scale of the economy measured by population size, $(B_{t+1} - B_t)/B_t = \tilde{g}(L_t, B_t)$, $\partial\tilde{g}/\partial L_t > 0$, $\partial^2\tilde{g}/\partial L_t^2 < 0$, and that there are decreasing returns to learning-by-doing, $\partial\tilde{g}/\partial B_t < 0$. The latter feature implies that asymptotically there is no (exponential) technology growth achieved from learning-by-doing, $\lim_{B\to\infty} \tilde{g}(L_t, B_t) = 0$. The learning-by-doing mechanism is appropriate to investigate technological and economic development for most of human history because technological advances were not (much) advanced by formally trained scientists before the industrial revolution (Mokyr 2002).

In an equilibrium with R&D, that is with $H_t^A > 0$, competitive R&D-firms employ H_t^A researchers to develop $A_t - A_{t-1}$ new blueprints and sell them at price p_t^A. Research output is given by

$$A_t - A_{t-1} = \delta_t H_t^A, \qquad \delta_t = \bar{\delta} A_{t-1}^{\phi} \left(1 + H_t^A\right)^{-\nu}. \tag{12}$$

Productivity of R&D, denoted by δ_t, is given to the single firm but depends at the aggregate level, positively on the number of already existing ideas (standing-on-shoulders effect,

[11] The steady-state and its comparative statics do not change if government revenue is (partly) used for public investment.

measured by ϕ, $0 \leq \phi < 1$) and negatively on R&D effort H_t^A (stepping-on-toes effect, measured by ν, $0 \leq \nu < 1$). The parameter $\bar{\delta}$ denotes general productivity in R&D. See Jones (1995) for a detailed motivation of the standing-on-shoulders and stepping on toes effects.[12]

Maximization of profits $p_t^A \delta_t H_t^A - w_t H_t^A$ implies that wages are given by $w_t = \delta_t p_t^A$ at an interior solution with positive R&D. Labor demand in research adds up with labor demand in final goods production to aggregate labor demand

$$H_t = H_t^A + H_t^Y. \tag{13}$$

At an equilibrium with R&D, wages in goods production and in R&D equalize such that $\delta_t p_t^A = (1-\alpha)Y_t/H_t^Y$. By inserting demand (10) into the goods price $p_t = r_t/\alpha$ and the result into profits, the free entry condition can be written as $p_t^A = \pi_t = \alpha(1-\alpha)Y_t/A_t$. Next, use these two equations for p_t^A to eliminate the price of blueprints and to arrive at labor demand in final good production

$$H_t^Y = \frac{A_t}{\alpha \bar{\delta} A_{t-1}^{\phi} \left(1 + H_t^A\right)^{-\nu}}. \tag{14}$$

The three Eqs. (12)–(14) can be solved for the three unknowns A_t, H_t^Y, and H_t^A given the predetermined variables A_{t-1} and $H_t = h_t L_t$. To see that H_t is pre-determined at time t note that $L_t = n_{t-1} L_{t-1}$ and recall that h_t is a function of w_{t-1} and h_{t-1} from (6).

The solution has to obey the constraint $H_t^Y \leq H_t$. The constraint becomes binding with equality if the solution of (12)–(14) would imply $H_t^Y > H_t$ and thus negative input in R&D. In this case aggregate human capital H_t (capturing the size of the market) is too low for R&D to be worthwhile. All human capital is allocated to final good production. Note that the corner solution depends on the abundance of *aggregate* human capital in the economy, $H_t = h_t L_t$. This means that the economy eventually leaves the corner solution if human capital per person or the number of workers is growing, i.e. when the size of the market, measured by H_t becomes sufficiently large.

The model thus suggests two potential triggers of R&D. In the first case R&D sets in before the fertility transition. In this case aggregate human capital is getting sufficiently large for R&D through an increasing workforce L_t. In the second case R&D sets in after the fertility transition, which means that it is triggered by increasing human capital endowment per worker h_t. In the calibration below we focus on the G-7, i.e. a case in which patented research (i.e. market R&D) sets in before the fertility transition. The implied rising wages subsequently initiate the onset of mass education and a fertility transition. The model, however, is general enough to allow as well for a calibration that leads to the reverse timing of events. This case appears to be more relevant for contemporary LDCs, in which the fertility transition sets in before market R&D becomes profitable and before the economy takes off with increasing TFP growth. Notice that a low productivity in R&D (a low $\bar{\delta}$) increases the gap between the fertility transition and the "productivity transition".

Turning towards the physical factors of production, we observe that population N_t grows at the fertility rate,

$$N_{t+1} = n_t N_t. \tag{15}$$

[12] In our notation Jones (1995) assumes $\delta_t = \bar{\delta} A_{t-1}^{\phi} (H_t^A)^{-\nu}$, which implies the counterfactual prediction of infinite productivity of the first person employed in R&D, $\lim_{H^A \to 0} \delta_t = \infty$. A situation without R&D would be impossible under this assumption. Our modification implies that productivity of the first person in R&D is finite.

Taking child rearing time into account the size of the workforce is given by $L_t = (1-\tau n_t)N_t$. Physical capital is assumed to depreciate fully within a generation such that next period's capital stock consists of this period's savings, $K_{t+1} = s_t N_t$. Inserting the solution for savings (2) and wages from (9) and (11) provides the evolution of aggregate capital,

$$K_{t+1} = \tilde{B}_t K_t^{\alpha} A_t^{1-\alpha} \left(H_t^Y\right)^{-\alpha} h_t L_t, \tag{16}$$

with $\tilde{B}_t \equiv B_t \beta(1-\alpha)/[(1-\tau n_t)(1+\beta+\eta)]$. This completes the model.

In the following we show why in modern economies lower fertility is associated with higher growth of TFP and income, along the steady state as well as along the adjustment path towards balanced growth. In the next section we focus on the steady state and its comparative statics. This allows us to prove the results analytically and to fully understand the underlying mechanisms. Section 6 then investigates adjustment dynamics for a numerically calibrated model. Compared to steady-state analysis, the investigation of adjustment dynamics has the disadvantage that results can no longer be proven analytically but it allows us to explore the historical take-off period to growth and to project the future in the medium-run.

4 The balanced growth path and its comparative statics

4.1 TFP growth and population growth

A balanced growth path (BGP) is defined as a steady state of the economy at which growth rates do not change and sectoral employment shares are constant. Constant population growth means that population L_t and workforce N_t grow at the same gross rate n_t, which is constant (above, below, or at replacement level) along the BGP. For any variable x, the growth rate is denoted by $g_{x,t} = (x_{t+1} - x_t)/x_t$ and its rate of change by $\hat{g}_{x,t} \equiv (g_{x_{t+1}} - g_{x_t})/g_{x_t}$. Balanced growth thus requires $\hat{g}_x = 0$ for $x = A, K, h, L$. We denote a growth rate of x along the BGP by g_x, i.e. by omitting the time index. Naturally, because of decreasing returns of learning-by-doing, asymptotically $g_B = 0$. Along the asymptotically attained BGP productivity growth is solely driven by market R&D. Using the fact that along the BGP H^A grows at the same rate as H and that g_A is constant we observe from (12) that

$$1 + g_A = (1+g_H)^{\frac{1-\nu}{1-\phi}} = \left[\frac{h_{t+1}}{h_t} \cdot n\right]^{\frac{1-\nu}{1-\phi}}, \tag{17}$$

in which the right hand side follows from the definition of aggregate human capital $H_t = h_t L_t$ and $L_{t+1} = n_t L_t$. Superficial inspection of Eq. (17) may lead to the conclusion that TFP growth and population growth are positively correlated, holding human capital growth constant. This view, however, disregards the interaction of fertility and education at the family level. Actually the quantity-quality trade-off suggests that population growth and human capital growth are correlated and that population growth cannot be varied "holding human capital growth constant". In other words, we would expect problems of multicollinearity if we put both population growth and human capital growth simultaneously as explanatory variables in a regression model for TFP.

This conclusion, however, does not apply for the pre-modern economy before the onset of the fertility transition. To see this, consider the corner solution in (3) and (4), in which there is no (mass) education, because family income is too low for formal education to be worthwhile. With $h_{t+1}/h_t = 1$ Eq. (17) reduces to

$$1 + g_A = n^{\frac{1-\nu}{1-\phi}}.$$

From this result we could indeed conclude that there is a unique positive association between productivity growth and population growth. The model has collapsed to an overlapping generations version of the well-known semi-endogenous growth model (Jones 1995). The problem is that, actually, there was a fertility transition. The results from the conventional R&D-based literature are thus hard to conceptualize as a steady-state phenomenon. The conventional model seems more appropriate for the period between the first and second industrial revolution, i.e. as a description of the *transitional* period during which there was already market R&D but not yet (mass) education. In the present model rising wages endogenously generate the onset of mass education. A positive association between R&D-based productivity growth and population growth is thus identified for a transitional period and not as a steady-state phenomenon.

After the onset of the fertility transition, i.e. after labor income surpasses z in (3) and (4), human capital and fertility are endogenous and *inversely* correlated via the quantity-quality trade-off. Evaluating (3) and (4) at a steady state with growth (i.e. for $w_t \to \infty$ and $h_t \to \infty$) we have that $n = \underline{n}$ from (5) and $h_{t+1}/h_t = \Delta_h$ from (7). Using this information we obtain steady-state productivity growth

$$1 + g_A = \left(\frac{\gamma A_E}{1+\beta+\eta}\right)^{\frac{1-\nu}{1-\phi}}. \tag{18}$$

Inspecting (18) we arrive at the main result of the paper.

Proposition 1 *A higher weight on child quality γ or a lower weight on child quantity η implies a higher rate of TFP growth and a lower rate of population growth along the balanced growth path.*

The proof computes from (18) that $\partial g_A/\partial\gamma > 0$ and $\partial g_A/\partial\eta < 0$ and uses Lemma 1. The result implies that for two otherwise identical economies TFP growth is higher in the economy in which parents put relatively less weight on child quantity, that is the one in which the fertility transition ends at a lower level of population growth. Notice that the result is independent from the size of n_t and holds as well when population growth $g_L = n_t - 1$ is negative.

For an intuition of the result recall the definition of aggregate human capital $H_t = h_t L_t$. Without congestion a positive effect of declining population on productivity requires that a higher preference for child quality exerts a stronger effect on human capital endowment per person of the next generation than on the number of persons such that h_t grows more than L_t falls. This is exactly what our model-parents provide. Inserting (5) and (7) into $H_{t+1}/H_t = n_t \cdot (h_{t+1}/h_t)$ we obtain the growth rate of aggregate human capital along the BGP, which depends positively on γ (and negatively on η)

$$g_H = \frac{\eta-\gamma}{(1+\beta+\eta)\tau} \cdot \frac{\gamma\tau A_E}{\eta-\gamma} - 1 = \frac{\gamma A_E}{1+\beta+\eta} - 1 \quad \Rightarrow \quad \frac{\partial g_H}{\partial\gamma} = \frac{A_E}{1+\beta+\eta} > 0.$$

The mechanism behind the result originates from the interaction in the budget constraint (and not from specifications of the utility function). To see this clearly consider a unit increase of e_t in company with a unit reduction of n_t such that total voluntary child expenditure $n_t e_t$ remains constant. This one-to-one quantity-quality substitution is not neutral. It sets free income $\tau w_t h_t$ because less time is needed for child rearing so that more time can be supplied on the labor market. The additionally earned income can be spent on current and future consumption and on further child expenditure e_t implying that the negative effect from reduction of fertility is smaller than the positive effect on human capital such that $H_t = h_t n_t$ rises. Because the mechanism arises from the budget constraint, we are confident that the result holds also for more general forms of the utility function as long as child quantity and quality are normal goods.

Turning towards the impact of time costs of children we see from (5) and (18) that a change of τ affects population growth but not productivity growth. Intuitively, rising costs of children lead to lower fertility and higher voluntary expenditure per child. For aggregate human capital $H_t = h_t L_t$ the negative effect through lower fertility and the positive effect via higher human capital growth per capita are exactly leveling each other such that $H_{t+1}/H_t = \gamma A_E/(1+\beta+\eta)$ is independent from τ and thus $\partial g_H/\partial\tau = 0$. The mechanics behind the result originate again from the budget constraint, but this time log-utility and its feature of balancing income and substitution effects plays a role as well. Higher child costs lead to lower child demand n_t and lower available income $(1-\tau n_t)w_t h_t$. With unchanged preferences income and substitution effect are balancing each other such that total expenditure $n_t e_t$ remains constant.[13]

4.2 Income growth and population growth

In order to examine the rest of the model, we begin with evaluating capital accumulation along the balanced growth path. From (16) we observe

$$\left(\frac{K_{t+2}}{K_{t+1}}\right) = \frac{B_{t+1}}{B_t}\left(\frac{K_{t+1}}{K_t}\right)^{\alpha}\left(\frac{H^Y_{t+1}}{H^Y_t}\right)^{-\alpha}\left(\frac{A_{t+1}}{A}\right)^{1-\alpha}\left(\frac{H_{t+1}}{H_t}\right).$$

Using that at the steady state $g_B = 0$ and $H^Y_{t+1}/H^Y_t = H_{t+1}/H_t$ and substituting g_H using (17) we obtain

$$1 + g_K = (1+g_A)^{1+\frac{1-\phi}{1-\nu}}. \tag{19}$$

The growth of capital and productivity are positively correlated at the steady state and capital growth is, ceteris paribus, large when there is a lot of congestion in R&D (when ν is small) and when knowledge spillovers are low (ϕ is small). For $\phi \to 1$ and $\nu = 0$ the model predicts that the capital stock grows at the rate of TFP growth.

Finally consider income per capita Y_t/N_t. At the steady state it grows at the same rate as income per worker $y_t = Y_t/L_t$ because fertility and time spent on children stays constant. From (11) we get

$$\frac{y_{t+1}}{y_t} = \frac{B_{t+1}}{B_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{H^Y_{t+1}}{H^Y_t}\right)^{1-\alpha}\left(\frac{K_{t+1}}{K_t}\right)^{\alpha}\left(\frac{L_{t+1}}{L_t}\right)^{-1}.$$

Evaluating the expression at the steady state and substituting g_K from (19) and g_H from (17) we arrive at income per capita growth according to (20)

$$1 + g_y = (1+g_h)(1+g_A) = \frac{\gamma\tau A_E}{\eta-\gamma}\left(\frac{\gamma A_E}{1+\beta+\eta}\right)^{\frac{1-\nu}{1-\phi}}, \tag{20}$$

[13] In an earlier version of this paper (Strulik et al. 2010) we considered the case that the stepping on toes effect in R&D depends on the number of workers instead of aggregate human capital input in R&D. In that case child rearing costs have an impact on productivity growth because duplication externalities result only from an increase of the number of workers but not from rising human capital per worker.

in which the right hand side followed after inserting g_A from (18) and Δ_h from (7). We arrive at the following result.

Proposition 2 *A higher weight on child quality in utility or a lower weight on child quantity implies a higher rate of growth of income per capita and a lower rate of population growth along the balanced growth path.*

The proof takes the derivatives of (20) with respect to γ and η and uses Lemma 1. Furthermore, taking the derivative of (5) and (20) with respect to τ proves the following result.

Proposition 3 *Higher child-rearing costs τ imply a higher rate of growth of income per capita and a lower rate of population growth along the balanced growth path.*

The reason is that higher child rearing costs induce a quantity-quality substitution. Aggregate human capital remains unchanged (see above) but GDP is divided only by the number of persons to get GDP per capita and thus GDP per capita benefits from the quantity-quality substitution.

It is instructive to compare R&D effort along the BGP with the earlier R&D-based growth models. Models of the first generation (Romer 1990; Aghion and Howitt 1992) predict constant TFP growth for a constant number of researchers. For this to be true the unpleasant knife-edge assumption $\phi = 1$ has to hold. Models of the second generation (Jones 1995; Segerstrom 1998) predict based on $\phi < 1$ that constant TFP growth is realized for a constant population share of researchers and positive population growth, implying the unpleasant result that constant economic growth requires a perpetually rising number of people employed in R&D. The present theory reconciles the earlier theories.

Proposition 4 *Along the balanced growth path constant TFP growth is associated with a constant share of the population working in R&D and constant R&D expenditure share of GDP. These results hold true for $\phi < 1$ irrespective of whether the number of people employed in R&D is rising, constant, or declining. If the population stays constant, constant TFP growth implies a constant number of workers engaged in R&D.*

For the proof denote the share of workers in goods production by $\theta_t \equiv H_t^Y/H_t$. Insert H_t^Y from (14) and the definition of δ_t to obtain

$$\frac{\theta_{t+1}}{\theta_t} = \frac{A_{t+1}}{A_t}\left(\frac{A_t}{A_{t-1}}\right)^{-\phi}\left(\frac{H_{t+1}^A}{H_t^A}\right)^{\nu}\left(\frac{H_{t+1}}{H_t}\right)^{-1}$$

At the steady state g_A is constant and H^A and H grow at equal rates. Using this information and g_H from (17) we get

$$\frac{\theta_{t+1}}{\theta_t} = \left[(1+g_A)\cdot(1+g_H)^{\frac{1-\nu}{1-\phi}}\right]^{1-\phi} = \left[(1+g_A)\cdot\frac{1}{(1+g_A)}\right]^{1-\phi} = 1,$$

implying that the share of workers in final goods production stays constant. In conclusion the share of workers in R&D stays constant.

For the second part of the proof, R&D expenditure is given by $R_t = w_t H_t^A$ and its share of GDP by $R_t/Y_t = w_t H_t^A/Y_t$. Insert wages from (9) to get $R_t/Y_t = (1-\alpha)H_t^A/H_t^Y$, which is constant since H_t^Y/H_t and H_t^A/H_t are constant along the steady state.

5 Adjustment dynamics: the take-off, the productivity reversal, and the future of R&D-based growth

In this section we investigate long-run adjustment dynamics towards balanced growth. We explore the onset of R&D-led growth, the take-off of mass education, and the productivity reversal. We consider an economy starting in pre-modern times, in a situation in which both non-negativity constraints, $w_t h_t - z \geq 0$ and $H_t - H_t^Y \geq 0$ are binding with equality. This means that initially there is neither market R&D nor mass education. Growth is solely driven by learning-by-doing as in Kremer (1993) and in Galor (2005). We calibrate the model such that market R&D sets in first (the first Industrial Revolution) and triggers, later on, the onset of (mass) education. Such a scenario seems not only to be in line with the historical evolution of England and Western Europe (Galor 2005) it also allows us to investigate a *transitional period* in which R&D growth is fueled solely by population growth; i.e. by the mechanism that is assumed to drive growth *at the steady state* according to the semi-endogenous R&D-based growth literature.

In a unified growth context, however, a period of R&D-based growth without education constitutes only a transitory phase and not a steady-state phenomenon. Eventually, rising income triggers education, and the demographic transition sets in. With rising human capital and declining fertility the economy converges towards the balanced growth path. While standard R&D-based growth theory would predict that economic growth declines with ongoing demographic transition because of declining population growth, the quality-quantity substitution operating in the present model allows for accelerating economic growth with declining population growth.

It can be argued that—because of knowledge spillovers—the country is a misleading unit of analysis for R&D-based growth. We try to accommodate this notion by calibrating the model to an average G-7, using population weights. This means that the US plays a relatively big role in the twentieth century and that the results are robust against reducing or increasing the group of countries by one country or another. The G-7 approximates a group of countries which can be thought of as producing leading-edge R&D-based research and which exchange knowledge with each other.

For the benchmark run we set the parameters β, γ, and η such that the model produces a savings rate of 0.2 along the BGP, a total fertility rate (TFR) of 3.8 children per couple ($n = 1.9$) before the demographic transition sets in, and a TFR of 1.78 at the end of the demographic transition. We thus assume that average fertility stays at the current level forever. Later on, we take up the UN projection about future fertility rates, including scenarios with fast and slow return of fertility to replacement level or above. The best match with the historical path of education is obtained when rearing a child takes 8.8 % of adult time ($\tau = 0.088$). This leads to the estimate $\beta = 0.31$, $\gamma = 0.14$, $\eta = 0.26$. After running the simulation we convert generational into annual data for better comparison with the actual historical time series by assuming that a generation takes 30 years.

We set $\alpha = 1/3$ and adjust $\phi = 0.35$, $\nu = 0.45$, and $A_E = 14$ to get the best fit of the empirical time series of fertility, income, and TFP growth during the twentieth century. We assume a learning-by-doing function $B_{t+1} - B_t = \mu L^{\lambda}$, set $\lambda = 0.4$ and $\mu = 0.005$ implying that income per worker grew at about 0.5 % per annum from 1600 to 1760 (in line with Broadberry et al. 2010). We set the remaining parameters $\bar{\delta}$, and $\bar{e}$ to get the speed of the fertility transition and the chronological distance between the onset of R&D-based growth and the fertility transition about right. A larger value of informal education $\bar{e}$ makes costly education less desirable and leads, ceteris paribus to a later and slower fertility transition. A larger value of $\bar{\delta}$ increases productivity in R&D. Ceteris paribus, it increases the time gap

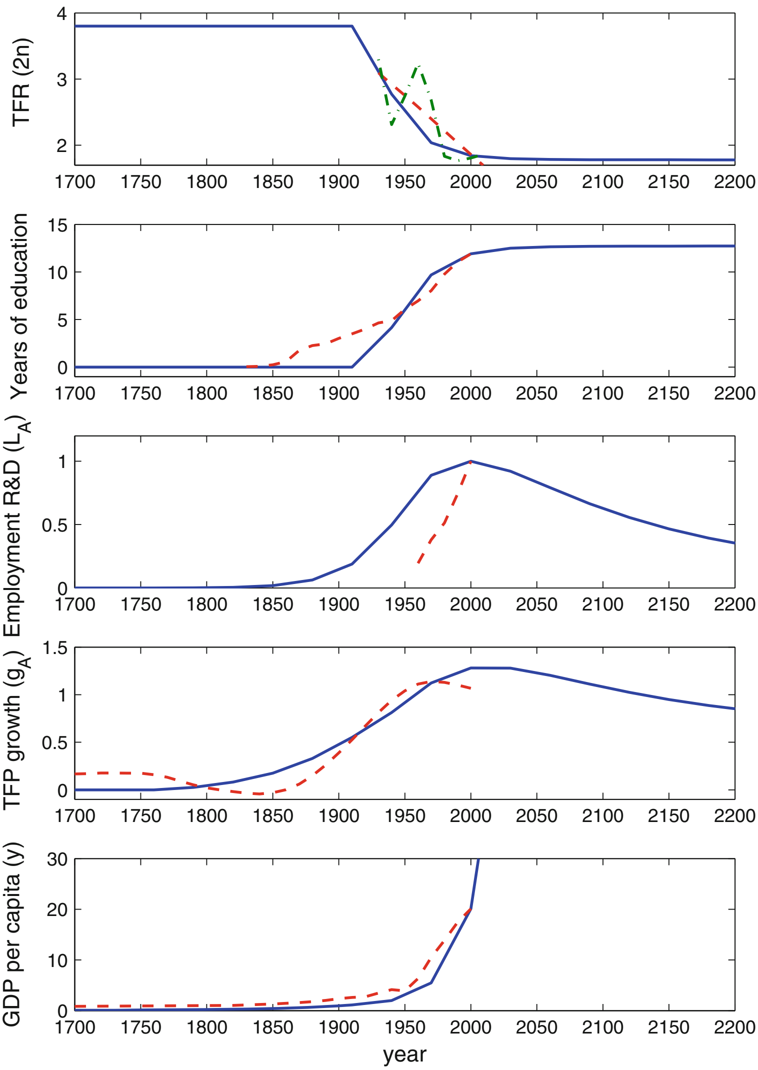

Fig. 3 The take-off to growth and long-run ddjustment dynamics. *Solid lines*: calibrated economy. *Dashed lines*: data for G-7 (country time series weighted by population size). TFP time series smoothed with Hodrick–Prescott filter (smoothing parameter 100). TFR: *red (dashed) line*: smoothed data (HP 100); *green (dash-dotted) line*: raw data. See text for details (Color figure online)

between the onset of R&D and the onset of the fertility transition. We get the best timing of the transition for $\bar{\delta} = 0.1$ and $\bar{e} = 0.12$. We set $A_0 = 50,\ L_0 = 0.001$ and $K_0 = 0.001$. We set the initial time to the year 1000 and set $B_0 = 0.01$ such that the onset of market R&D happens in the year 1760.

Solid lines in Fig. 3 show the implied adjustment dynamics. The implied path of growth of R&D output g_A is compared with the smoothed time series of average TFP growth in the G-7. The model predicts correctly that TFP growth is much higher during the twentieth century than in the centuries before and gets the magnitude of TFP growth about right. The first panel in Fig. 3 shows the evolution of fertility. In order to compare with the real data we have doubled the number of children predicted by our unisex model to get the number of children per couple of adults, which we depict jointly with the average TFR in the G-7 countries. The green (dashed-dotted) line shows the actual data and the red (dashed) line shows the smoothed long-run trend that appears after applying the Hodrick–Prescott filter. The trend eliminates the baby boom after World War II. The onset of the fertility decline is predicted by the model for the year 1910, which coincides with the actual onset of the fertility decline in England and captures the average onset of the decline in the G-7 reasonably well (see Fn. 6). The model gets the speed of the fertility transition (reflected by the slope of the time series) about right, implying that it correctly predicts the actual TFR in 2010. In the benchmark scenario we assume that fertility follows this trend and levels off at a TFR of 1.78 at the end of the century. We discuss alternative scenarios below.

In order to compare the model predictions with the actual data we convert schooling expenditure, measured relative to labor income, $e/(wh)$, into years of education. We normalize such that the schooling effort in the year 2000 corresponds to 11.9 years of education, the observed G-7 average in the same year. The onset of mass education, according to our model, happens jointly with the fertility transition, a feature that is shared with the standard theory of unified growth (Galor 2005). With respect to the G-7 this means that education is predicted to start rising about 2 generations too late, as shown in the second panel from above. For the purpose of this paper, however, the second half of the twentieth century is more interesting, because then (in 1970) fertility crosses the replacement level and the population starts to decline. For the 1960–2000 period the model prediction virtually coincides with the G-7 data. The model predicts that the huge growth of education is a unique phenomenon of the twentieth century. For the twenty-first century and after, education effort (years of schooling) is predicted to level off. This implies that the growth rate of human capital converges from above towards about 1.1 %.[14]

The third panel in Fig. 3 shows employment in research, measured as the number of researchers $L_A \equiv H_A/h$. The model predicts a very gradual increase in the eighteenth century, which really gains momentum during the twentieth century, when birth rates are falling. The dashed trajectory shows the number of scientists and engineers in the G-7. For better comparison we have normalized predicted and actual times series such that both coincide at 100 in the year 2000. Compared with the data, the model gets the growth of employment in R&D (represented by the slope of the curve) about right. The steep increase of R&D, however, happens about a generation too early. The increase of R&D-employment is predicted to level off in the late twentieth century and then to decline gradually. It is thus possible, as an off-steady-state phenomenon, that the population declines (since 1970 according to the model) while the scale of R&D employment is relatively stable. As a consequence the model predicts almost constant TFP growth for the first half of the twenty-first century.

The bottom panel in Fig. 3 shows the evolution of income per capita. Income starts to increase somewhat with the onset of R&D-based growth and then really takes off with increasing education. The model matches the evolution of income per capita for the average G-7 reasonably well.

[14] The coincidence of the onset of fertility decline and eduction is a consequence of the simple structure of the model. Taking child mortality into account and the fact that all children incur rearing costs but only surviving children get an education decouples the two transitions and leads to an interim phase in which education takes off but net fertility and population growth rise due to falling mortality. Another channel decoupling the two transitions would be compulsory schooling.

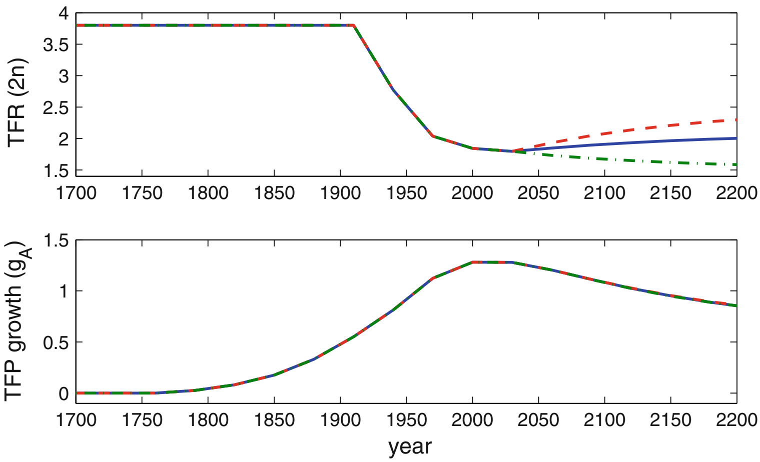

Fig. 4 UN population projections and R&D-based productivity growth. Economy calibrated as in Fig. 3. After the year 2000 the parameter η is adjusted to match the UN TFR projections. *Solid (blue) lines*: UN-medium; *dashed (red) lines*: UN-high, *dashed-dotted (green) lines*: UN-low (Color figure online)

6 The future of R&D-based growth: comparing outcomes of UN-fertility predictions

It could be hypothesized that fertility rates in contemporaneous fully developed countries have reached a trough or will reach a trough in the near future and that fertility will recover towards replacement or an even higher level in the more distant future. The model as it is offers no endogenous mechanism by which fertility may recover from below replacement level. But undershooting fertility behavior can be conveniently discussed through parametric changes. Specifically, we re-calibrate the model to capture the UN (2011) fertility projections for the G-7 in the twenty-first century and re-asses the future growth perspectives in the medium-run.

We calibrate the UN fertility projections through adjustment of attitudes towards children by adjusting η, the weight of children in utility. We calibrate η_∞ to match the UN projections for the long-run and compute convergence as $\eta(t) = \eta_\infty + \rho^{t-2010} \cdot [\eta(2010) - \eta_\infty]$. We calibrate η_∞ such that two times the associated fertility rate $n(\eta_\infty)$, according to (4), coincides with replacement level fertility. We set $\rho = 0.5$ to approximate the path of the UN fertility projection. The blue (solid) line in Fig. 4 shows the implied evolution of the TFR and productivity growth. Red (dashed) lines in Fig. 4 show the model's prediction under the UN high-fertility projection. This means that η_∞ is adjusted such that $2n$ converge to 2.5 during the twenty-first century. Green (dash-dotted) lines show the outcome under the UN low-fertility projection, assuming that $2n$ converge towards 1.5. For the twenty-first century it is hard to discern any differential impact on TFP growth for three different UN scenarios. The predicted productivity growth at the steady state is 0.65 under the low fertility scenario and 0.60 under the high fertility scenario. This means that the effects of higher population growth and—according to the quantity-quality trade-off—lower human capital growth are almost balancing each other with respect to aggregate productivity growth. The differential impact on growth of income per capita is, of course, larger because high fertility increases the denominator of Y/L. The model predicts 2.4 % growth of income per capita under the low fertility scenario and 1.0 % for the high fertility variant.

7 Conclusion

In this paper we have integrated R&D-based innovations into a unified growth framework. We have shown how the consideration of an endogenous, microfounded evolution of fertility and education modifies some central findings of the earlier R&D-driven growth theory. While earlier models (in the spirit of Romer 1990 or Jones 1995) predicted that population growth is positively associated with economic growth, or even—in the Jones case—essential for having economic growth in the long run, our micro-founded theory predicts that the erstwhile positive association between population growth and productivity growth turns negative in the twentieth century. The modified R&D-based growth theory is thus less easily refuted by the available data for the twentieth century.

Since we have maintained all central elements about the firms' side from R&D-based growth theory it is clear that the new results originate from household behavior. The basic mechanism is generated by the interaction of child quality and quantity in the households' budget constraint and is observed independently from the specification of preferences, which makes us confident that our results are robust against a sophistication of the households' utility function.

Specifically, a substitution of child quantity n by child quality (i.e. expenditure on education) e that keeps total child expenditure $e \cdot n$ constant sets free parental time, which can be used to earn extra income. The additional income is partly spent on education such that overall child expenditure rises more strongly than child quantity falls. At the macro side of the economy this trade-off means that human capital per person h increases more strongly than the number of persons L falls such that total available human capital $h \cdot L$ increases. Given that human capital is the driving force in R&D this entails higher R&D output and higher R&D-based growth.

We have calibrated the model for an average G-7 country and shown that the negative association between population growth and productivity growth is not only observable along the balanced growth path but also during the adjustment phase in the twentieth century. The unified growth model has also produced novel insights about the timing of the onset of mass education and R&D-based growth. It is capable of explaining how a first Industrial Revolution, brought forward by tinkerers, initiated mass education and a second Industrial Revolution, after which R&D is produced by formally trained scientists. These details about the timing of long-run growth cannot be explained by the so far available growth theories because they neglect either R&D-based growth (unified growth theory) or the micro-foundation of fertility and human capital accumulation (conventional R&D-based growth theory).

Taking the quality-quantity trade-off into account allowed us to draw a much less grim conclusion about economic growth in the near future than suggested by the conventional R&D-based growth literature. Our brighter assessment of the future does not depend on the assumption of constant returns to education. More generally we could have considered human capital production according to $h_{t+1} = g(e_t, h_t)$ and could have allowed for decreasing returns with respect to e_t. Essential for perpetual growth are constant returns with respect to h_t, that is a linear *intergenerational* transmission of human capita. This means that the current generation is capable to transmit its knowledge times a multiplier larger than one to the next generation. While it is impossible to say whether such a process of knowledge transmission can be sustained forever, it is in any case easier conceivable than a perpetually growing population. Human capital is a metaphysical entity measured in value-units whereas population is a physical entity bounded by physical constraints as, for example, space on earth. To see the intergenerational multiplier of the value of knowledge at work compare, for example, the value of knowledge acquired by a university study of medical science now and

100 years ago. A plausible extension of the model could take into account that efficiency of the education technology A_E is also time-varying and positively influenced by the currently available productive knowledge.

Instead of venturing forth into the domain of speculation about the distant future of humanity we would like to emphasize that our model is a metaphor to explain economic growth in the past, the present, and the near future (say within the limit of UN fertility projections). In the recent past, we observed high TFP growth in line with high growth of human capital and low and increasingly negative population growth, and we expect these trends to continue for a while. In this respect the main message delivered by the model is an optimistic one: the fact that fertility is below replacement level and population is declining should not be threatening. It promises high productivity growth when R&D is advanced by educated scientists and engineers.

In the very long-run it is likely that fertility below replacement level and negative population growth run against physical and economic limits. This insight could have been another reason why the UN recently reconsidered its 2008 projections; assuming now adjustment to replacement level from below at the end of the twenty-first century (see also Myrskyla et al. 2009). With respect to TFP growth our calibration for the G-7 suggests that there is hardly any differential impact between the different UN population scenarios. The reason is that a quantity-quality substitution at the family level balances the effects of population growth and human capital growth on aggregate human capital. For income per capita growth, however, it is more favorable when aggregate human capital growth is driven by increasing human capital per person instead of an increasing number of persons.

The model has been calibrated to accommodate the UN fertility projections by adjusting preferences. Alternatively, Strulik and Weisdorf (2008) have developed a unified growth model of a two-final-goods economy, which endogenously and independently from preferences and child costs produces convergence towards a stationary population in the very long-run and which predicts undershooting and negative population growth during the twenty-first century as a transitional phenomenon. Combining these ideas with the present work is a challenging task for the future.

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8 Data appendix

8.1 List of countries for Table 1

Albania, Algeria, Angola, Argentina, Armenia, Australia, Austria, Azerbaijan, Bangladesh, Belarus, Belgium, Benin, Bolivia, Botswana, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Zaire, Congo, Costa Rica, Cote d'Ivoire, Cyprus, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Fiji, Finland, France, Gabon, Gambia, Georgia, Ghana, Greece, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, Hungary, India, Indonesia, Iran, Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Kuwait, Kyrgyzstan, Laos, Latvia, Lesotho, Liberia, Libya, Lithuania, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mauritius, Mexico, Moldova, Morocco, Mozambique, Myanmar, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Romania, Russia, Rwanda, Saudi Arabia, Senegal, Sierra Leone, Singapore, Slovak Republic, Somalia, South Africa, South Korea, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syria, Taiwan, Tajikistan, Tanzania, Thailand, Togo, Trinidad, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Arab Emirates, United Kingdom, USA, Uruguay, Uzbekistan, Venezuela, Vietnam, West Germany, Yemen, Zambia, Zimbabwe

8.2 Data sources and construction of aggregated historical time series

We focus on the following countries (G-7): Canada, France, Germany, Italy, Japan, United Kingdom and United States. The data set spans from 1650 to 2000 and was constructed as a population-weighted average of the individual countries' time series. In so doing we relied on the following sources:

(1) From 1650 until 2000 we used per capita GDP and population size obtained from Angus Maddison's website at http://www.ggdc.net/MADDISON/oriindex.htm, see also Maddison (2001) for a detailed description.
(2) Data on TFP growth rates for the United Kingdom from 1650 to 2000 are from Madsen et al. (2010), TFP growth rates for the United States between 1870 and 2000 are from Gordon (1999), and TFP growth rates for Canada, France, Germany, Italy and Japan are from Baier et al. (2006). For Canada the series covered the years 1880 to 2000, for France the years 1860 to 2000, for Germany the years 1890 to 2000, for Italy the years 1870 to 2000 and for Japan the years 1900 to 2000.
(3) Years of schooling data are from Baier et al. (2006). The corresponding series were available from 1830 to 2000 for the United Kingdom, from 1850 to 2000 for France, from 1860 to 2000 for Italy, from 1870 to 2000 for Canada and the United States and from 1890 to 2000 for Japan.
(4) The data on R&D-employment between 1950 and 2000 in France, Germany, Japan, the United Kingdom and the United States stem from the website of Charles I. Jones (http://www.stanford.edu/~chadj/Sources50.asc, see also Jones, 2002, for a detailed description). We extended these series from 1994 to 2007 by using the growth rates of the number of researchers in the corresponding countries implied by OECD data (OECD 2012).
(5) Fertility projections until 2100 were obtained from the United Nation's Population Division (United Nations, 2011). The data is available at http://esa.un.org/unpd/wpp/index.htm.
(6) As another robustness check we included the total fertility rates of the Human Fertility Database (2012). The data is available at http://www.humanfertility.org/cgi-bin/main.php.

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